# STOCHASTIC OPTIMAL CONTROL MODELING OF DEBT CRISES

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## Abstract

What is an optimal or a sustainable external debt - for a country, region or sector? How should one monitor and evaluate debt to preclude a crisis? We use stochastic optimal control/dynamic programming to derive an optimal debt. The deviation of the actual from the optimal will serve as a Warning Signal of a crisis. There is a correspondence between Hamilton-Jacobi-Bellman equation of Dynamic Programming and the static Mean-Variance (M-V) analysis in finance. A graphic analysis of M-V is helpful to explain the implications of DP. An explicit example is the US Agricultural debt crisis.

JEL Code: C61, D81, D9, F34.

Keywords: stochastic optimal control, debt, international finance, US agricultural crisis, Mean-Variance analysis, Hamilton-Jacobi-Bellaman equation.

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## *1. Types of debt crises*

Debt crises have occurred in the South-East Asian, Latin American and African countries. Economies have borrowed either to finance government deficits, or investment/ capital formation as in South-East Asia. The debt, denominated in either domestic or foreign currency, may either be short-term that must be repaid with interest at maturity, or it may be long-term which just has to be serviced regularly. There are several sources of uncertainty, which may be positively or negatively correlated: the real return on capital and the variable real interest rate. Since it is not possible to predict either the return on capital or the interest rate, default has occurred frequently.

The issues discussed in this paper are as follows. (a) How much of a debt is too much? What is an optimal or a sustainable external debt - for a country, region or sector? (b) How should one monitor and evaluate debt to preclude a crisis? (c) We use stochastic optimal control/dynamic programming to derive an optimal debt. The deviation of the actual from the optimal will serve as a Warning Signal of a crisis. (d) There is a correspondence between Hamilton-Jacobi-Bellman equation of Dynamic Programming and the static Mean-Variance (M-V) analysis in finance. A graphic analysis of M-V is helpful to explain the implications of DP. This paper is an application of Fleming-Stein (2004), Fleming-Pang (2003), and is a generalization of the Merton model. The analysis in Fleming-Stein (2001) has been applied by Stein-Paladino (2001) to crises involving short-term debt.

An explicit example used in this paper is the US Agricultural debt crisis, because the empirical data correspond closely to the appropriate mathematical variables. Figure 1 plots three variables concerning the US agricultural sector: the ratio of interest payments/value added, equity = capital less debt, and the delinquency rate on loans to commercial banks. During the prosperous years 1972-80 land values rose and, using the capital gains as collateral, farmers incurred debt to buy more land which produced more capital gains. The boom generated by capital gains was unsustainable. From 1980-86, farm equity fell by by three standard deviations with a rise in bankruptcies and defaults on loans. The shaded area refers to the debt crisis. Using available information, what warning signals should have alerted the farm and banking sectors to the impending crisis?

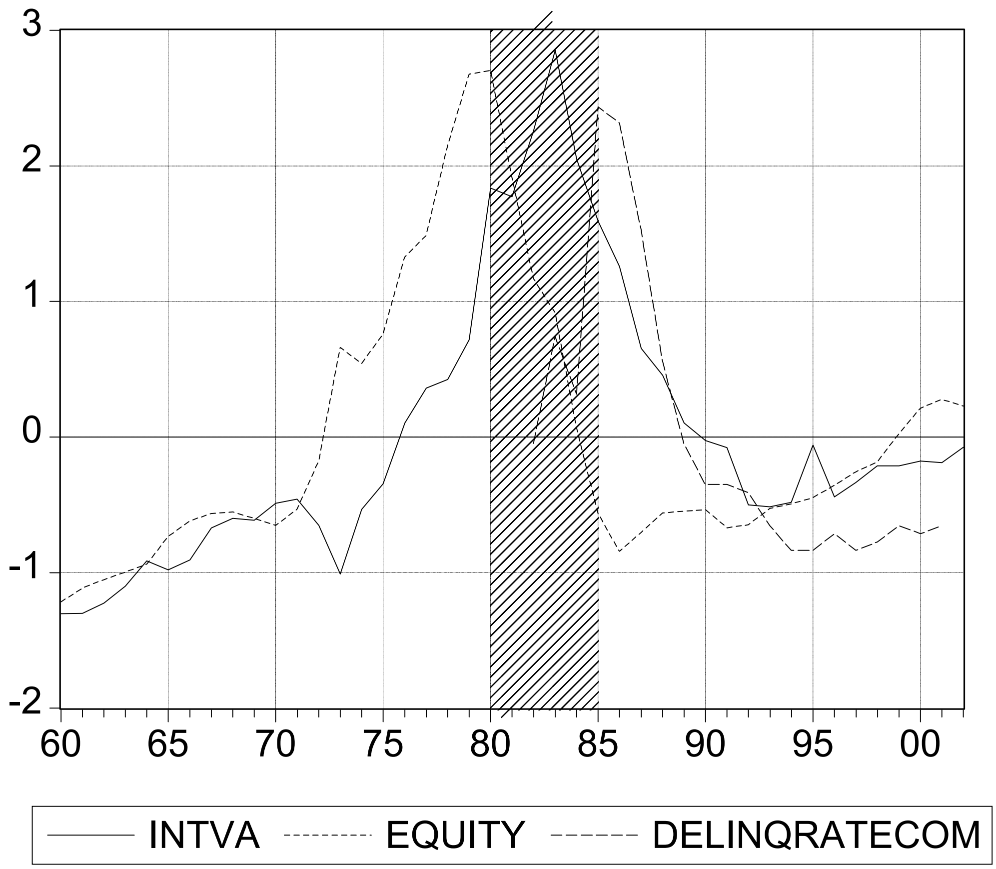


Figure 1. US Agriculture 1960 - 2002. INTVA = total interest payments/value added. EQUITY = assets - debt . DELINQRATECOM = delinquent farm loans to commercial banks, percentage of outstanding loans. Normalized variables: (variable - mean)/standard deviation.

## *2. A Prototype model*

The model underlying the optimization is summarized in the equations in BOX 1. The variables are real, measured in terms of goods produced. This benchmark model is sufficiently general to be applicable to almost any economy. The crucial features are that the *fundamentals* - the real return on capital and the variable real interest rate - are described by equations of Brownian motion. There is an expected return on capital but the actual return does not revert to a constant mean; and a similar situation exists for the real rate of interest. It is not possible to predict either the value of the return on capital or the interest rate. These equations are used to derive benchmarks of *Rational Expectations* (RE) *optimal* performance, *conditional upon the real fundamentals: the real return on capital and the interest rate*.

The *actual* debt deviates from the RE optimal because market participants base decisions upon anticipated capital gains that are not based upon these fundamentals. The unsustainable capital gains resulting from these anticipations generate speculative bubbles, like the dot.com in the US and the real estate boom in South-East Asia. The deviation between the actual and the optimal debt is a bubble that bursts when subjected to shocks. In part 5 below, we explain the farm debt crisis and provide warning signals.

The performance criterion to be maximized is the expected present value of the utility of consumption, equation (1). The maximum value is V(X), where X is the initial net worth. The utility function selected is HARA, equation (1a) where risk aversion is $(1-\gamma) > 0$, $\gamma \neq 0$. The HARA utility function has two great advantages. First, it implies that we may focus upon ratios, such as consumption/net worth $c = C/X$, debt/net worth $f = L/X$ and capital/net worth $k = K/X$. Second: the HARA assumption lowers the dimension of the dynamical system, and the model can be solved analytically. Otherwise, the DP model must solved using numerical values and a computer. When $\gamma = 0$, risk aversion is $(1-\gamma) = 1$, and the utility function is (1b) the logarithmic function. An infinite time horizon is selected with an arbitrary discount rate of $\delta > 0$. The effective length of the horizon is inversely related to the discount rate. A high discount rate implies a short horizon. If $\gamma < 0$, there is no mathematical need to have a discount rate.

**BOX 1**.

EQUATIONS OF THE STOCHASTIC GROWTH MODEL

(1) $V(X) = \max_{c,f} E_w \{ \int_0^\infty U(C(t))\, e^{-\delta t}\, dt \}$

(1a) $U(C) = (1/\gamma)C^{\gamma}_t$, $\gamma < 1$, $\gamma \neq 0$; (1b) $U(t) = \ln C_t$, $\gamma = 0$, C = consumption

(2a) $dL_t = r_t L_t dt + (C_t + I_t - Y_t)dt$ L = debt, $f = L/X$

(2b) $C_t dt = Y_t dt - r_t L_t dt - I_t dt + dL_t > 0$ $c = C/X$

(3) $K_t = P_t N_t$ K = capital, Y = value added

(4) $Y_t\, dt / P_t N_t = b_t\, dt = b\, dt + \sigma_b\, dw_b$, $dw_b = \varepsilon_b \sqrt{dt}$, $\varepsilon_b \sim N(0, 1)$ iid

(5) $r_t L_t dt = r L_t dt + \sigma_r L_t dw_r$; $dw_r = \varepsilon_r \sqrt{dt}$, $\varepsilon_r \sim N(0, 1)$ iid interest payments

(5a) $L_t < hR_0 e^{\eta t}$ lending constraint

(6) $E(\varepsilon_b\, \varepsilon_r) = \rho$, $1 \geq \rho \geq -1$.

(7) $dP_t/P_t = \mu\, dt + \sigma_p dw_p$. P = relative price of "land" N

(7a) $E(dw_p, dw_b)) = 0$, $E(dw_p, dw_r) = 0$

(8) $X_t = K_t - L_t = P_t N_t - L_t$ net worth = equity

Equations (2a) and (2b) are the same. In (2a), the change in the debt $dL_t$ is the sum of the debt service $r_t L_t$ at interest rate $r_t$, plus consumption $C_t$ plus investment $I_t$ less income $Y_t$, over a period of length dt. Equation (2b) puts consumption on the left hand side. It is income less debt service less investment over the period plus new borrowing. For an economy, the sum of value added is GDP or income, denoted $Y_t$.

The production function is equations (3)-(4). Equation (3) states that capital $K_t$ is the product of a physical quantity $N_t$, "land", times the $P_t$ relative price of land/price of output. Equation (4) states that the productivity of capital, ratio $b_t\, dt = Y_t\, dt / K_t$ of gross value added to capital, is described by a stochastic process. The productivity of capital is the sum of a deterministic term $b$ dt plus a stochastic term $\sigma_b\, dw_b$. Call $b$ the mean return on investment. The stochastic part of the growth of value added term $\sigma_b\, dw_b$ results from changes in prices of output relative to the purchased inputs of materials, physical productivity such as output/acre, and variations in demand.

Equation (5) describes the stochastic servicing of the debt $r_tL_t$ over the short period dt. The real rate of interest over the short period $r_t$ dt is the sum of a deterministic term - the mean r dt - plus a stochastic term, which has a variance $\sigma_r^2$ dt over the period. The stochastic part results from variations in monetary policy, the business cycle or, when an external debt in denominated in foreign currency, from changes in the exchange rate. The demand for loans by the entire agricultural sector is $L_t$. Constraint (5a) states that $L_t$ cannot exceed the loans that the banking system is willing to supply. The latter is a multiple h > 0 of the reserves $R_0e^{\eta t}$ which grow at exogenous rate $\eta$. This constraint is important in explaining in section (5.2) below why bubbles burst and debt crises occur.

Each disturbance, to either the return on investment or to the real rate of interest, is time independent. Equation (6) states that the two disturbances, to the growth rate and to the interest rate, may be correlated, either positively or negatively. This correlation occurs by matching up one variation in $dw_b$ with one in $dw_r$.

Figure 2 graphs the two basic stochastic variables in the farm sector. The measured return per annum on real farm assets $b_t$ = real gross value added/real farm assets is denoted GVACAP; and the interest rate per annum $r_t$ = interest expense/farm debt is denoted INTDEB. Neither variable reverts to a constant mean. The correlation between them switches between positive and negative.

Net worth or equity $X_t$, equation (8) is defined as capital $K_t$, from (3), less debt $L_t$. We constrain the optimization to the case where net worth X(t) > 0. This constraint excludes "Ponzi schemes", where the economy borrows to service the debt ad infinitum.

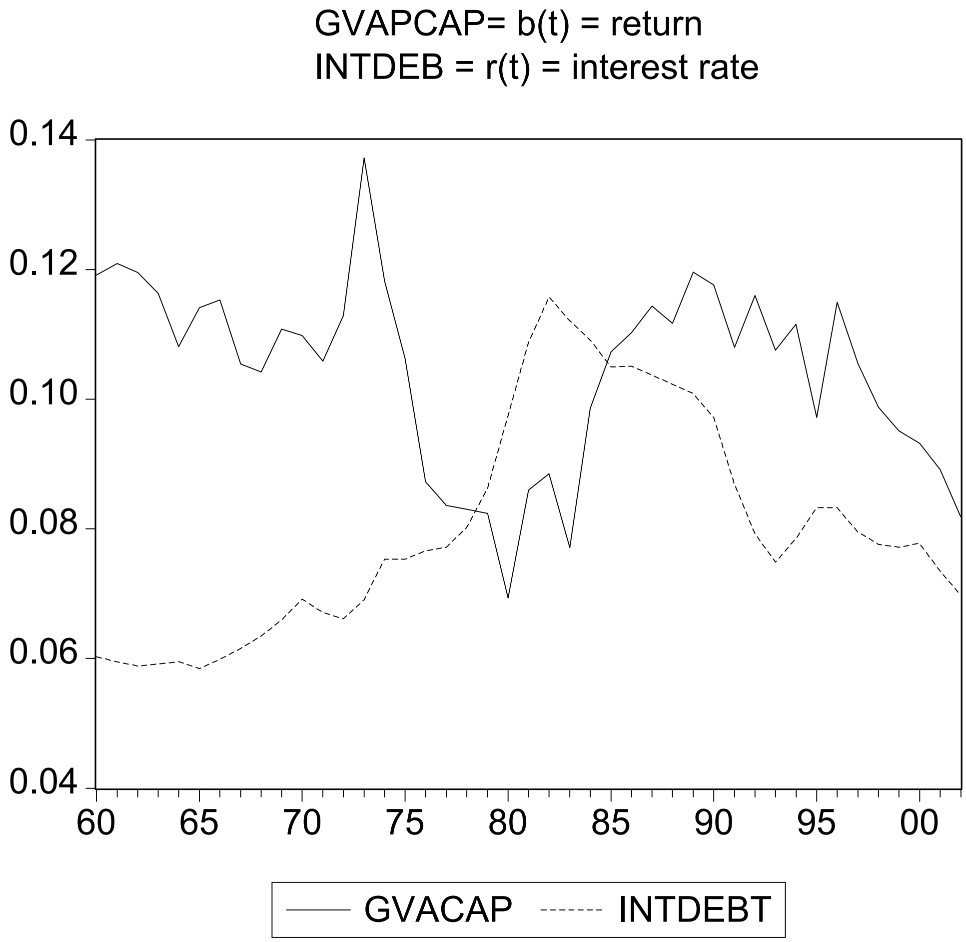


Figure 2. U.S. Farm sector. Return = $b_t$ = gross value added/value farm assets = GVACAP. Interest rate = $r_t$ = interest expenses/debt = INTDEBT

The change in the value of assets $dK_t = d(P_tK_t)$ is equation (9). The first term $P_tdN_t = I_t\, dt$ is investment at market prices, and the second term $(N_tP_t)(dP_t/P_t)$ is the capital gain (or loss) resulting from the rise (fall) in the price of the physical asset - $N_t$ called "land" - relative to the price of output.

(9) $dK_t = P_tdN_t + (N_tP_t)(dP_t/P_t) = I_t\, dt + K_t(dP_t/P_t)$

In many debt crises - such as US agriculture, South-East Asia, or the dot.com stocks - there is a "bubble", defined as the case where the capital gains equation (7) are independent of the movements of the fundamentals $b_t$ and $r_t$.

The market assumes that there is an upward trend $\mu > 0$ in the relative price of "land", plus a Brownian motion term $\sigma_p dw_p$. Both the Brownian motion (BM) term $\sigma_p dw_p$ and the trend term $\mu$ are independent of the other two BM terms in BOX 1, as described by equation (7a). In the case where there are "rational expectations" (RE), the market price of the asset will be closely linked to the fundamentals $b_t$ and $r_t$ and $\mu = 0$. Our standard of optimal performance is the rational expectations case.

We have assumed that the return $b_t = Y_t\, dt / P_t N_t = Y_t\, dt / K_t$ = real gross value added/real value of assets is described by stochastic process equation (4), which is graphed as return GVACAP in figure 2. This formulation was selected because data are available for Y and K = PN, and Y/K can be described by Brownian motion. On the other hand, Fleming and Pang (2003) assume that the return real gross value added/acre = $Y_t\, dt / N_t$ is described by a stochastic process. In the RE case, both forms are identical, but the results differ in the case where $\mu > 0$.

*We show below that the optimization based upon* $\mu > 0$ *leads to an unsustainable situation - a debt crisis. The deviation between the debt/net worth held in case where* $\mu > 0$ *and the debt/net worth held in the case of rational expectations case* $\mu = 0$ *provides us with Warning Signals of a debt crisis.*

### *3. Dynamic Programming Solution*

In this section we derive the dynamic programming solution for the optimal debt. Then it is shown how the inter-temporal dynamic programming *DP* solution can be given a mean-variance *M-V* interpretation. We then compare the optimal market debt when $\mu > 0$ with the rational expectations RE solution when $\mu = 0$.

The *state variable* is net worth X(t) defined in equation (8). It is capital $K_t$ less $L_t$ debt. The change in net worth is equation (10).

(10) $dX_t = dK_t - dL_t$

Derive equation (11) by using (9) for $dK_t$ and (2a) for $dL_t$. The HARA function implies that consumption is proportional to net worth $C_t = cX_t$ and the debt is also proportional to net worth $L_t = fX_t$, (Fleming-Stein, 2004). The ratio f = L/X is debt/net worth. From (8), capital/net worth $k_t = K_t/X_t = 1 + f_t \geq 0$. Equation (11) is the stochastic differential equation. The first set of terms in brackets is deterministic and the second is stochastic.

(11) $dX_t/X_t = [(b-c) + (b-r)f + (1+f)\mu]\,dt + [(1+f)\sigma_b dw_b - f\sigma_r dw_r + (1+f)\sigma_p\,dw_p]$

There are two controls, $u = (f,c)$. One control is $c = C_t/X_t > 0$ the consumption ratio. The second control is the debt ratio $f = L_t/X_t > -1$. A negative debt is net financial assets. It is assumed that the debt can be varied instantly and with no cost. This assumption is too strong and will be relaxed in subsequent research. The optimization (1) is subject to the dynamic equation (11) and to the constraints $C_t > 0$, $X_t > 0$.

Given the nature of the uncertainty, the controller cannot anticipate the future. The admissible controls are chosen using any information known up to time t. We therefore consider the controls that enter as feedback functions of the state $X_t$. The Hamilton-Jacobi-Bellman dynamic programming equations (12a) and (12b) are based upon the dynamics of the change in net worth, equation (11). See Fleming-Soner (1992).

(12a) $\delta V(X) = \max_u [G^u V(X) + (1/\gamma)(cX)^\gamma]$, $u = (f, c > 0)$.

(12b) $G^u V(X) = [(b-c) + (b-r)f + (1+f)\mu]XV_x$

$+ (1/2)[f^2\sigma_r^2 - 2f(1+f)\rho\sigma_r\sigma_b + (1+f)^2(\sigma_b^2 + \sigma_p^2)]X^2V_{xx}$

With the HARA utility function, we may write the value function as equation (13) where the constant $A > 0$ is to be determined from (12) and (13).

(13) $V(X) = (A/\gamma)X^\gamma$

Using equation (13) and its derivatives in (12) we derive the DP equation (14) where $V^*(f,c)$ is defined in (14a,b,c).

(14) $\delta/\gamma = \max_{c,f} \{(1/\gamma)c^\gamma/A + V^*(f,c)\} = \max_{c,f} \{(1/\gamma)c^\gamma/A + M(f,c) - (1-\gamma)R(f)\}$

(14a) $V^*(f,c) = [M(f,c) - (1-\gamma)R(f)]$

(14b) $M(f,c) = [(b-c) + (b-r)f + (1+f)\mu]$

(14c) $R(f) = (1/2)[f^2\sigma_r^2 - 2f(1+f)\rho\sigma_r\sigma_b + (1+f)^2(\sigma_b^2 + \sigma_p^2)]$

The optimal debt/equity f is independent of the optimal consumption ratio. Define the *market optimal debt/net worth ratio* $f_s$ as one where $\mu > 0$, when there are speculative capital gains, and define the *Rational Expectations optimal* $f^*$ as the special case where $\mu = 0$. The optimal ratio is stated in equation (15), where $\sigma^2$ and f(0) are defined in equations (15a), (15b) respectively. The ratio $\theta = \sigma_r/\sigma_b$ = standard deviations of the interest rate/ rate of return and ratio $\omega = \sigma_p/\sigma_b$ = standard deviations of the capital gain / rate of return. The market optimal debt/net worth ratio $f_s$ is a linear function of the expected net rate of return including the trend capital gain $(b + \mu - r)$. The expected real rate of return is *b*, and the expected real rate of interest is $(r - \mu)$, where $\mu$ is the expected capital gain. As shown in section (5.2) below, the expected capital gain $\mu > 0$ is unsustainable. The Rational expectations case sets $\mu = 0$ and the optimal debt/net worth f* is equation (15-RE). The slope is the reciprocal of risk aversion $(1-\gamma)$ times risk $\sigma^2$ = variance $(b - r + dp/p)$ in equation (15a). The market debt/equity ratio $f_s$ will exceed the RE value $f^*$ if the expected trend inflation of asset prices $\mu > 0$. Equation (15c) relates to constraint (5a). The demand for loans in the RE case $f^*X^*_t$, where $X^*_t$ is the net worth in the RE case, does not exceed the loans that banks are willing to make.

Optimal debt/net worth $f_s$, Rational Expectations Case f*

(15) $f_s \in \text{argmax}\ [M(f,c) - (1-\gamma)R(f)] = (b + \mu - r)/(1-\gamma)\sigma^2 - f(0)$

(15-RE) $f^* = (b - r)/(1-\gamma)\sigma^2 - f(0)$

(15a) $\sigma^2 = \text{var}\,(b - r + dp/p) = (\sigma_b^2 + \sigma_r^2 + \sigma_p^2 - 2\rho\sigma_b\sigma_r) = \sigma_b^2\,(1 + \theta^2 + \omega^2 - 2\rho\theta)$

(15b) $f(0) = (1 + \omega^2 - \rho\theta) / (1 + \theta^2 + \omega^2 - 2\rho\theta)$

(15c) $f^*X^*_t < h\, R_0 e^{\eta t}$

Equation (15) is a generalization of Merton's equation for the optimal ratio of risky assets/net worth. In Merton's model, the rate of interest is deterministic so $\theta = 0$. There is only one component of the return on the risky asset, either $b_t$ or $dp_t/p_t$. Let it be $b_t$ so that $\omega = 0$. (We could have chosen $dp_t/p_t$ and ignored $b_t$). Then, equation (15) for the optimal risky assets/net worth $k = 1 + f$ is equation (16). This is exactly Merton's equation.

(16) $k^* = 1 + f^* = (b - r)/(1-\gamma)\sigma_b^2$

The meaning of the intercept term f(0) is discussed in the Mean-Variance section below.

### *4. A "mean-variance" (M-V) interpretation*

The Tobin-Markowitz mean variance (M-V) analysis is the cornerstone of much of the work in the field of investment/portfolio allocation analysis. It is extensively used in the agricultural finance literature to evaluate risk for agricultural firms. The M-V analysis is based upon a static two period model of portfolio choice between "safe" and "risky" assets, whose great virtue is that it yields clear and operational results. Our model in BOX 1 seems to be quite different. Growth is endogenous over an infinite horizon and there is risk on both the debt and on capital.

W*e show how the inter-temporal dynamic programming equations (14- 15) can be given an interpretation in the traditional static two-period "mean-variance" portfolio choice model.* The DP equation (14) can be expressed in several ways. The first term $[(1/\gamma)c^{\gamma} /A]$ is the utility of present consumption, where term A > 0 is to be determined from the solution. The second term V*(f,c) can be interpreted in equation (14a) as *Expected M-V utility.* It is equal to the **M**ean, equation (14b), less the product of risk aversion $(1-\gamma) > 0$ and **R**isk, equation (14c). The **M**ean is the expected percentage change in net worth $E(dX_t/dt)/X_t$ in equation (11) if there were no risks. In the deterministic case, when one selects the controls (c,f) we know for certain that net worth X and consumption cX grow at rate M(f,c). There are also stochastic components to the change in net worth. We must also consider the risk terms $R(f,c) = (1/2)\ \mathrm{var}\ [dX_t/X_t]$, which is one half of the variance of the growth in net worth. Risk contains the control debt ratio f, variances and co-variances. Hence the DP equation (14) can be interpreted as (17). Maximize the sum of the utility of current consumption plus V* = M-V expected utility: a linear combination of a mean M and $(1-\gamma)R$ a risk R times positive risk aversion.

(17) $\delta/\gamma = \max_{c,f}$ {utility current consumption + [Mean - (risk aversion)(Risk)]}

*Equation (17) shows that the maximization with respect to the optimal debt/net worth from the DP equation can be given a M-V interpretation, since debt is only in the V* term.* A negative debt is a positive financial asset position.

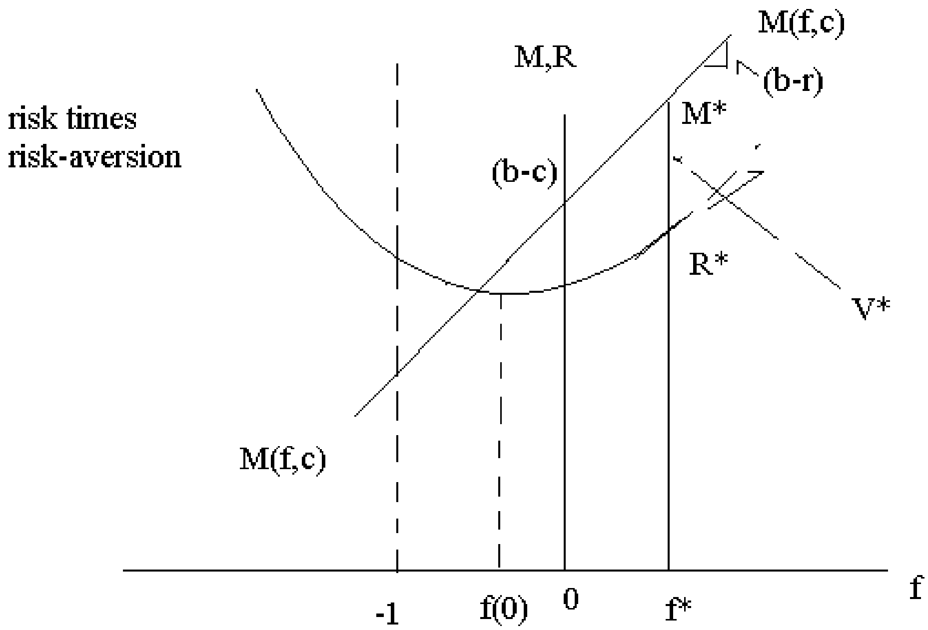


Figure 3. Mean-Variance Interpretation of the DP equation for optimal debt/net worth f* in the Rational Expectations case, where $\mu = 0$.

A graphic discussion of the determination of f* the optimum debt/net worth is described in figure 3, for the Rational Expectations case where there is no expected speculative capital gains. The DP equation requires that we select a debt/net worth ratio f that maximizes the "mean-variance expected utility" $V^* = M(f,c) - (1-\gamma) R(f)$. The mean M(f,c) is a linear function of f the debt/net worth. The slope of the Mean function dM/df = (b - r) is the expected return less the expected interest rate. Intercept (b-c) is the expected return less the consumption ratio. Variations in the consumption ratio only affect the intercept and not the slope of the Mean function. *Therefore optimal debt/net worth is independent of the consumption ratio.*

The uncertainty concerns the risk R(f), which is a quadratic function of the debt/net worth, and is defined in equation (14c). In figure 3 quadratic risk function R(f) reaches a minimum at $f = f(0)$, which is equation (15b) - the intercept term in the optimal debt/equity equation (15). To minimize risk when $f(0) < 0$, the firm should be a creditor.

There is a big difference between the M-V interpretation of the DP equation and the static two-period M-V analysis. For example, in the DP interpretation/ RE case denoted by asterisks, optimal debt $L_t^* = f^*X^*_t$. Net worth $X^*_t$ varies according to stochastic differential equation (11) when $\mu = 0$. As $X^*_t$ varies, the debt $L^*_t$ must be varied instantaneously to maintain the ratio f* to net worth $X^*_t$. On the other hand, in static M-V analysis (Robison et al), a rise (decline) in net worth implies that, while holding constant the level of risky assets, the debt should be decreased (increased). Thus *inter-temporal DP analysis can be given a M-V interpretation, but one cannot go from static M-V to inter-temporal optimization.*

**5. Application to farm debt crisis**

5.1. Accumulation of debt in the prosperous years

An application of the DP technique to explain the US farm debt crisis of the early 1980s (see FDIC, 1997) may serve as a model how to monitor the debt of any country or region. The agricultural crisis has been selected because there are available data that closely correspond to the theoretical variables.

Agriculture flourished in the 1970s. Farm exports grew rapidly and along with the domestic inflation farm incomes reached all-time highs. These factors produced capital gains on farm assets. Credit was readily available. Real interest rates $(r - \mu)$ were low and farmers used the rising value of farm assets as collateral for loans. Farmers would purchase farm real estate with moderate down payments and, after the value of the newly purchased land increased, would use the increased equity to buy additional farm land with minimal downpayments. Higher levels of real estate debt were supplemented by debt to finance machinery and equipment. The speculation in land produced capital gains $\mu > 0$ and raised X the market value of equity. Figure 1 shows how the ratio of interest payments/value added = INTVA = debt burden, grew as the farm EQUITY rose. By 1979, the debt burden was almost 3 standard deviations higher than it was in 1973. Lenders were not concerned since equity rose by as much, due to the capital gains.

5.2 Crisis: collapse of the bubble

The bubble started to burst in 1979 and the depression continued through the decade of the 1980s. In the fall of 1979, the Federal Reserve undertook a restrictive monetary policy and interest rates $r_t$ rose drastically. The resulting appreciation of the US dollar reduced foreign demand for US agricultural products. The decline in foreign demand was exacerbated by the debt crisis in the less developed countries. Farm exports declined by 40% from 1981 to 1986, at a time when productive capacity had increased. The decline in demand is a decline in $b_t$. Figure 1 shows how EQUITY fell from 1980-86 by about 3 standard deviations, and farm delinquency rates on commercial bank loans rose drastically. We contrast the Rational Expectations case $\mu = 0$ with the market bubble $\mu > 0$, and derive Warning Signals of an impending crisis.

Defaults will occur at time T when the economy cannot service the debts without a decline in consumption, described by equation (25), based upon (2a) above even when investment is zero. The key to the crisis or collapse of the bubble is that the interest payments are growing rapidly, there is a negative shock to the return on capital $b_t$, a positive shock to the interest rate $r_t$ and there is a *constraint* on new bank lending $dL_t$.

(25) $C_T dt = (b_T K_T - r_T f_s X_T) - I_T + dL_T$

As a result of the bubble $\mu > 0$, the ratio of debt/net worth is $f_s$, net worth is $X_T$ and interest payments are $r_T L_T = r_T f_s X_T$. The cash flow at crisis time T = 1980 is the first term in parentheses: value added $Y_T = b_T K_T$ less interest payments $r_T L_T = r_T f_s X_T$. At time T= 1980, there was a negative shock to the productivity of capital $b_T$, and a positive shock to the interest rate $r_T$, which produced a severe decline in cash flow. Consumption would fall unless the farmers obtain new loans $dL_T$ to service the debt. This Ponzi scheme will not last for long for the following reason.

The expectation of the debt $L_t = f_s X_t$ in the bubble case exceeds that in the Rational Expectations case (denoted by asterisks), since $f_s > f^*$ and $X > X^*$. When $\mu > 0$, the expectation of the debt that would exist at time T is $E(L_T) = f_s E[X_T] = f_s X_0 \exp [((b - c) + (b - r)f + (1+f)\mu) T]$. The expected net worth is derived from stochastic differential equation (11) using the proof in Øksendal, pp. 60-61.

The lending constraint is assumed to be satisfied in the RE case, equation (15c). However, when the bubble $\mu > 0$ is significantly high, such that $((b - c) + (b - r)f_s + (1+f_s)\mu) > \eta$, the lending constraint will eventually be violated. The bubble case is the left hand expression and the RE is the right hand expression.

(26) $[E(L_T) = f_s E(X_T)] > h R_0 e^{\eta T} > [E(L^*_T) = f^* E(X^*_T)]$

The banks/lenders are constrained to lend no more than multiple h of their reserves which grow at rate $\eta$.As the speculative bubble continues, loans rise towards the maximum that the banks are willing to finance. The availability of funds for new loans declines, particularly when cash flows decline. Even though interest rates may not reflect it, banks decrease the availability of credit. The decline in the availability of new loans $dL_T$ means that the third term in (25) declines; and it may even turn negative. The decline in all three terms in (25) implies that: To service the debt, consumption must decline. This will lead to defaults and hence a debt crisis. *This scenario also describes many of the major international debt crises.*

5.3 Warning Signals

W*arning signals WS will be based upon the difference between the actual debt/net worth and the sustainable RE optimal f*. Similarly, the WS will be the difference between the interest payments/value added and the RE optimal based upon f*.*

We base our estimates of the means *b* and *r* upon available information. The US Department of Agriculture, Economic Research Service (ERS), Agricultural Income and Finance, Farm Income and Balance Sheet Indicators contains data for: $Y_t$ = value added, $K_t$ = value of farm assets = capital , $L_t$ = debt, $r_tL_t$ = interest expenses, and $(K_t - L_t)$ = equity = net worth, all measured in constant dollars on a base 1996=100. The sample period is 1960-2002.

The interest rate on the debt $r_t$ = INTDEB is a weighted average of the interest rates $r_tL_t/L_t$ . Five-year moving averages of gross real value added/real capital = GVACAP = $Y_t/K_t$ and INTDEB from year T-5 to the present year T, are used to estimate the means. The mean net return is a five year moving average of real gross value added/real farm assets less interest rate, denoted by $(b-r)_5$ , measured as percent per annum. *This variable is based upon available information.* Both $(b-r)_5$ and the variance of the net return are slowly changing over time.

Ideally, we would like our warning signal $WS_t$ to be the difference between the actual debt/net worth = $L_t/X_t$ and the RE optimum f* in equation (15-RE).

(27) $WS_t = L_t/X_t - f^*_t = L_t/X_t - [m(b - r)_5 - f(0)]$, where $m = 1/(1-\gamma)\sigma^2$

As the difference increases, the debt burden $r_tL_t/Y_t$ rises and the probability of a debt crisis increases, see equation (26). The measure of risk aversion is arbitrary, and the variance differs among sub-periods. Therefore, we use two warning signals, evaluated when $m = 1/(1-\gamma)\sigma^2 = 1$, and $f(0) = 0$. In (28a), we measure the deviation of the actual debt/equity ratio from the moving average of the net return. This is labeled WARNING.

(28a) $\text{WARNING} = L_t/X_t - (b\text{-}r)_5$

Warning signal WARNING2 in (28b) is the deviation of interest payments/value added $r_tL_t/Y_t$ from the moving average of the net return.

(28b) $\text{WARNING2} = r_tL_t/Y_t - (b\text{-}r)_5$ .

Both warning signals are graphed in figure 4, where the variables are normalized, (variable - mean)/standard deviation. Each warning signal is measured in units of standard deviations to facilitate a comparison and provide orders of magnitude. The normalization is why the evaluation at m = 1 and f(0) = 0 is not a problem.

The farm debt crisis occurred during the period 1980-88, the shaded area, when defaults and delinquency ratios rose drastically. On the basis of this figure we see the usefulness of the warning signals WS. From 1965-75, there were no warning signals - the "green light" was on. From 1973-77, the two WS rise by one standard deviation - the "amber light" is on. By 1979, the WS has risen to two standard deviations from 1975: the "red light" is on. In 1980, the crisis occurs.

From 1975-83 the 5-year moving average net return was declining, and from 1979-84 the actual net return was negative. The actual numbers are as follows.

| | debt/equity | mean net return (b-r) | interest expense/value added |
|---|---|---|---|
| 1975 | 20% | 4.55% | 11.8% |
| 1985 | 29.8 | -1.87 | 22.48 |

The optimal debt/equity ratio and interest expense/value added should have been declining rather than rising. Our analysis, based upon Dynamic programming, correctly predicts a debt crisis in the shaded region and correctly predicts tranquil periods pre-1979 and post 1990.

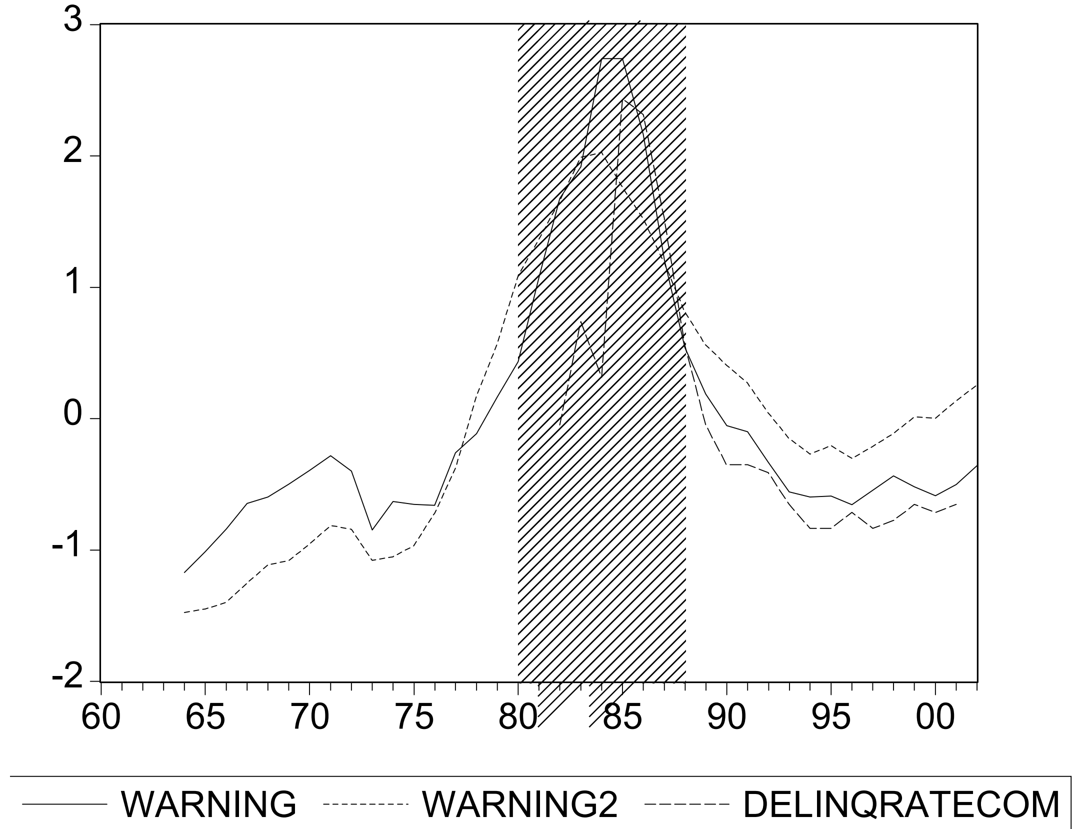


Figure 4. WARNING = debt/equity - retvaintd5 = $(L/X)_t - (b - r)_5$ , WARNING2 = interest payments/value added $r_tL_t/Y_t - (b - r)_5$ , DELINQRATECOM = delinquency rate on commercial bank loans for agriculture

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